Calculus sample problems IV for final test in Fall Semester

1. Show that if a function f is monotonic in $[a,b]$, then f is integrable.
2. If $f(x)=\int_0^x(\int_0^t \underline{\sin^{100} s}ds)dt$, find $f''(x)$.
3. $\int_{-2}^{2}|x|dx=$
4. $\int_1^2(1+x^{-\frac{2}{3}}+x^{\frac{2}{3}})dx=$
5. $\int_0^2\min(x,1)dx=$
6. $\int_0^1\frac{1}{\sqrt{1+x}}dx=$
7. $\int(\sin\underline{5x})(\cos\underline{2x})dx=$
8. The sequence $a_1,a_{2,}a_3\cdots\to b$ means that for every $\varepsilon>0$, there exists an N (depending on ε), such that $\forall n>N\Rightarrow|a_n-b|<\varepsilon$. Show that the limit of a sequence is unique if it exists.
9. Express $\lim_{n\to\infty}(\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{n+2n})$ as a definite integral.
10. Find the area of the region bounded by $y=\sqrt{x}$ and the line passing through $(0,0)$ and $(4,2)$.
11. Find the volume of a sphere of radius r.
12. Show that $\ln(ab)=\ln a+\ln b$ if $a>0$, $b>0$.
13. $D\sin\sqrt{1+\ln\sin x}=$
14. $\int\frac{1}{x(\ln x)(\ln\ln x)}dx=$
15. $\int_1^2 2^x dx=$