Calculus sample problems II for midterm test in Fall Semester

1. Let $\varepsilon > 0$ be given, find $\delta > 0$ such that $\left| x^2 - 4 \right| < \varepsilon$ whenever $0 < \left| x - 2 \right| < \delta$.

2. $\lim\limits_{x \to 1} \dfrac{\sqrt{3 + \sqrt[3]{x}} - 2}{x - 1} =$

3. $\lim\limits_{x \to \infty} \sqrt[4]{x^4 + x^3} - x =$

4. Let $f(x) = \begin{cases} x \sin \dfrac{1}{x}, & x \neq 0 \\ c, & x = 0 \end{cases}$. Find the value of c such that f is continuous at $x=0$.

5. Let $f(x) = \begin{cases} x^2 \sin \dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Does $f'(0)$ exist? Prove your answer.

6. If $f(x) = x^2$, $g(x) = 2x$, find $(fg)'(x)$, $(f'(g(x)))$, $(f \circ g)'(x)$ and $(g \circ f)'(x)$

7. Find the derivative of the function defined by $f(x) = \dfrac{x(\sin x^2)}{2x + 1}$.

8. Find $D \sin(\cos(\sin x))$.

9. Let $x + y + \sin(xy^2) = 1$, find $\dfrac{dy}{dx}$.

10. Find the equation of the tangent line at the point $(1,1)$ to the graph of the equation $x^2 + xy + y^2 = 3$.

11. Show that $\left| \sin x - \sin y \right| \leq \left| x - y \right|$.

12. Let A be the area of a circle of radius r. If r is changing with respect to time t, find the relation between $\dfrac{dr}{dt}$ and $\dfrac{dA}{dt}$.

13. Sketch the graph of the function $y = f(x) = x(x-4)^3$ and find possible points of inflection.

14. Find the maximum value of the function $f(x) = \dfrac{\left| x \right|}{\sqrt{(x-4)^2 + 9}}$, $x \in R$.

15. Find the maximum volume of those right circular cylinders inscribed in a sphere of radius 1.