Answeres to calculus sample problems for final test in Fall Semester

1. Let f be increasing and P_n be a partition of $[a,b]$ into n subintervals of equal length. It is easy to check that $T(P_n)-S(P_n)=(f(b)-f(a))(\frac{b-a}{n})$, which can be less than any given $\varepsilon>0$ if n is chosen big enough.

2. $f'(x)=\int_0^x \sin^{100} s ds$, and $f''(x)=\sin^{100} x$.

3. $\int_{-2}^2 |x|dx=\int_{-2}^0 |x|dx+\int_0^2 |x|dx=\int_{-2}^0 (-x)dx+\int_0^2 xdx=4$

4. $\int_1^2 (1+x^{-\frac{2}{3}}+x^{\frac{2}{3}})dx=(x+3x^{\frac{1}{3}}+\frac{3}{5}x^{\frac{5}{3}})\Big|_1^2$ =(2-1)+3($2^{\frac{1}{3}}-1)+\frac{3}{5}(2^{\frac{5}{3}}-1)$

5. $\int_0^2 \min(x,1)dx=\int_0^1 xdx+\int_1^2 1dx=\frac{1}{2}+1=\frac{3}{2}$

6. $\int_0^1 \frac{1}{\sqrt{1+x}}dx=\int_1^2 u^{\frac{-1}{2}}du=2\sqrt{u}\Big|_1^2=2(\sqrt{2}-1)$

7. $\int (\sin 5x)(\cos 2x)dx=\int \frac{1}{2}(\sin 7x+\sin 3x)dx=\frac{-1}{14}\cos 7x+\frac{-1}{6}\cos 3x$ +c

8. Assume that $a_1,a_2,a_3\cdots\to b_1$ and $a_1,a_2,a_3\cdots\to b_2$ .If $b_1\neq b_2$, choose

$\varepsilon=\frac{|b_1-b_2|}{2}$ and we can find N_1, N_2 such that $|a_n-b_1|<\varepsilon$ for n>N_1 ,

and $|a_n-b_2|<\varepsilon$ for n>N_2. Let t=N_1+N_2. Then we have $|a_t-b_1|<\varepsilon$ and

$|a_t-b_2|<\varepsilon$. This implies $|b_1-b_2|\le|a_t-b_1|+|a_t-b_2|<2\varepsilon=|b_1-b_2|$, which is a contradiction. Thus $b_1=b_2$.

9. $\lim_{n\to\infty}(\frac{1}{n+1}+\cdots+\frac{1}{n+2n})=\lim_{n\to\infty}(\frac{n}{n+1}+\cdots+\frac{n}{n+2n})(\frac{1}{n})=$

$\lim_{n\to\infty}(\frac{1}{1+\frac{1}{n}}+\cdots+\frac{1}{1+\frac{2n}{n}})(\frac{1}{n})=\lim_{n\to\infty}\sum_{i=1}^{2n}(\frac{1}{1+\frac{i}{n}})(\frac{1}{n})=\int_1^3 \frac{1}{x}dx$

10. $A=\int_0^4 (\sqrt{x}-\frac{x}{2})dx=\frac{4}{3}$

11. $V=2\int_0^r (\sqrt{r^2-x^2})^2\pi dx=\frac{4}{3}\pi r^3$

12. Let $f(x)=\ln ax-\ln x, x>0$. We have $f'(x)=0$ .Hence f is constant. Since $f(1)=\ln a$, it follows that $f(x)=\ln ax-\ln x=\ln a$, i.e. $\ln(ax)=\ln x+\ln a$.

13. $D\sin\sqrt{1+\ln\sin x} = (\cos\sqrt{1+\ln\sin x})\ (\frac{1}{2})(1+\ln\sin x)^{\frac{-1}{2}}(\frac{\cos x}{\sin x})$

14. $\int \frac{1}{x(\ln x)(\ln\ln x)}dx = \int \frac{1}{u}du = \ln|u| = \ln|\ln\ln x| + \mathrm{c}$

15. $\int_1^2 2^x dx = \int_1^2 e^{x\ln 2}dx = \frac{1}{\ln 2}2^x\Big|_1^2 = \frac{2}{\ln 2}$